



FIG. 10. The function  $(W - W_I)T^{-1}$  versus  $T$  for Si as a means of determining the extrapolated lattice thermal resistivity at high temperatures.

Here  $B_U$  has the approximate value<sup>43</sup> of

$$B_U \sim (\hbar\gamma^2/2\pi m\theta v^2) \exp(-\theta/3T), \quad (5)$$

where  $\gamma$  = Grüneisen's constant assumed to be equal to 2, and  $m$  = average mass of a single atom. For boundary scattering the relaxation time for diffuse scattering of the phonons off the walls of the crystal is simply

$$\tau_B^{-1} = v/L, \quad (6)$$

where  $L$  is the average diameter of the crystal. For Si these various relaxation times have been included in the  $K$  expression given in Eq. (1) using the following values:  $v = 6.4 \times 10^5$  cm/sec,  $\theta = 674^\circ\text{K}$ ,  $V_0 = 1.99 \times 10^{-23}$  cm<sup>3</sup>,  $\Gamma = 1.65 \times 10^{-5}$ , and  $L = 0.44$  cm. The results of the calculations for various combinations of relaxation times are shown in Fig. 9 for Si. It can be seen that all three relaxation times must be used in order to obtain agreement between theory and experiment. The theoretical curve with all three scattering processes included gives a maximum  $K$  of 56 W/cm deg at 28°K for Si compared to the observed maximum of 39 W/cm deg at 25°K. The fit at lower  $K$  values is rather better. In view of the simple model used for deriving Eq. (1) such agreement is considered reasonable. A similar sort of analysis has been carried out by other authors<sup>6,37</sup> for Ge at low

temperatures, where the agreement is even better. Some improvement in the fitting of the theory to experiment can be made by including the effects of both the transverse and longitudinal phonon branches and the effects of phonon dispersion.<sup>44</sup> These added complications will not be considered here. The primary conclusion to be drawn from the low-temperature results for  $T < 300^\circ\text{K}$  is that phonons are the only significant carriers of the thermal energy, and they are scattered by other phonons, isotopes, and the crystal boundaries. Other carriers of thermal energy do, however, become important for  $T > 300^\circ\text{K}$ .

#### INTERPRETATION OF HIGH-TEMPERATURE MEASUREMENTS

##### Lattice Thermal Conductivity

In the present context high temperature means  $T > 300^\circ\text{K}$ . In this region it is necessary to consider photons and electrons and holes as well as phonons as possible carriers of thermal energy. We shall assume that these various carriers interact only weakly so that their contributions to the thermal conductivity are additive.

<sup>44</sup> M. G. Holland, Phys. Rev. 132, 2461 (1963).